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CONTROL LIMITS FOR VARIABLE FRACTION DEFECTIVE – P CHART

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ABSTRACT

In Industry one is faced with two kinds of problems (i) to check whether the process is confirming to standards laid down and (ii) to improve the level of standard and reduce variability, consistent with cost considerations. Shewart's control charts assume that the fraction defectives are a constant. But due to several uncontrolled factors, the fraction defective varies. Hence in this paper the fraction defective is allowed to vary stochastically and the system solution is obtained by solving stochastic differential equations. In this study, a design procedure of p-chart for variable fraction defective is presented and control limits are derived using stochastic differential equations.

KEYWORDS: P-Chart, UCL, LCL, Variable Fraction Defective, Stochastic Differential Equations

INTRODUCTION

A stochastic differential equation (SDE) is a differential equation in which one or more terms is a stochastic process, thus resulting in a solution which itself a stochastic process.SDE are used to model diverse phenomena such as fluctuating process parameters or physical system subject to fluctuations. Typically SDE incorporate white noise which can be thought of as the derivative of Brownian motion (or the Weiner process); However it should be mentioned that other types of random fluctuations are possible such as jump process.

Lawrence. C. Evans has given an introduction to stochastic differential equations. H. Sussmann (1977) has developed an interpretation of stochastic differential equation with ordinary differential equations, which depends on the sample point. L. Arnold (1974) has given stochastic differential equations; theory and applications. K. L. Chung (1975) has developed "Elementary probability theory with stochastic process". Control charts for attributes can be used for quality characteristics,(i)which can be observed only as attributes by classifying an item as defective or non defective ie. Confirming to specifications or not (ii) which are actually observed as attributes even though they could be measured as variables (eg).go and non go gauge test results. The widely applied control chart for attributes are

- Control chart for fraction defectives(p-chart) or the number of defectives (np or d-chart)
- Control charts for the number of defects per unit(c-chart).

Jafari Jozani M, Mirkamali S. J (2011) have developed control charts for attributes with maxima nominated samples. Aebtarn .S, Bougila .N (2011) have introduced an empirical evaluation of attribute control charts for monitoring defects. Perry M.B, Pignatiello J.J (2011) have made an attempt on, 'estimating the time of step change with Poisson CUSUM and EWMA control charts.' Institute of Industrial engineers (1999) have developed Improved p-charts to monitor process quality. Ryan T. P, Schwertman (1997) have given optimal limits for attribute control charts. Woodall, William H (1997) have developed control charts based on attribute data.

DERIVATION OF NEW CONTROL LIMITS FOR p-CHART FOR VARIABLE FRACTION DEFECTIVES

The Stochastic Differential Equation is

$$\begin{cases} \dot{X}(t) = b\big(X(t)\big) + B\big(X(t)\big)\xi(t) \quad (t > 0) \\ \qquad X(0) = x_0 \end{cases}$$

Let p(t) denote the fraction non-conformities at a time t≥ 0. A Standard model assumes that $\frac{dP}{P}$, the relative change of defectives, evolves according to the SDE

$$\frac{dP}{P} = \mu dt + \sigma dW$$

In other words,

$$\begin{cases} dP = \mu P dt + \sigma P dW \\ \quad P(0) = p_0, \end{cases}$$

Where p_0 is the initial non-conformities of the process. Using $It\hat{o}'s$ formula we have derived the model for the variable process fraction defective as,

$$p(t) = p_0 e^{\sigma W(t) + (\mu - \frac{\sigma^2}{2})t}$$

This solution is embodied in the control limits of p chart to control the production process.

Therefore if the fraction defective is variable in nature, then the new control limits are,

$$\text{UCL} = \overline{p(t)} + 3\sqrt{\frac{\overline{p(t)}(1-\overline{p(t)})}{n}}$$

$$\text{CL} = \overline{p(t)}$$

$$\text{LCL} = \overline{p(t)} - 3\sqrt{\frac{\overline{p(t)}(1-\overline{p(t)})}{n}}$$

Where $\overline{P(t)} = \overline{p_0 e^{\sigma W(t) + (\mu - \frac{\sigma^2}{2})t}}$

Hence,

$$\text{UCL} = \overline{p_0 e^{\sigma W(t) + (\mu - \frac{\sigma^2}{2})t}} + 3\sqrt{\frac{\overline{p_0 e^{\sigma W(t) + (\mu - \frac{\sigma^2}{2})t}}\left(1 - \overline{p_0 e^{\sigma W(t) + (\mu - \frac{\sigma^2}{2})t}}\right)}{n}}$$

$$\text{CL} = \overline{p_0 e^{\sigma W(t) + (\mu - \frac{\sigma^2}{2})t}}$$

$$\text{LCL} = \overline{p_0 e^{\sigma W(t) + (\mu - \frac{\sigma^2}{2})t}} - 3\sqrt{\frac{\overline{p_0 e^{\sigma W(t) + (\mu - \frac{\sigma^2}{2})t}}\left(1 - \overline{p_0 e^{\sigma W(t) + (\mu - \frac{\sigma^2}{2})t}}\right)}{n}}$$

Where $\overline{p(t)} = \sum \frac{p(t)}{n}$

ARL of P-Chart for Variable Fraction Defective

For statistical control, the control chart parameters are determined by specifying the average run length (ARL) for the in-control state (controlling the probability of false alarms) and the ARL of one or more out-of-control process conditions (controlling the detection capability). Saniga (1991gives the relationship between the probabilities for type I and type II errors and the ARL and the average time to signal (ATS). Assuming the process is in control and letting α represent the probability of a type I error (false alarm) the ARL in the in-control state can be expressed as; $ARL = \frac{1}{\alpha}$

Assuming the process is out of control and β represent the probability of a type II error (failure to defect), the ARL in the out of control state can be expressed as;

$$ARL = \frac{1}{1-\beta}$$

The Operating Characteristic Function and Average Run Length Calculations

The operating characteristic (or OC) function of the Variable fraction non confirming control chart is a graphical display of the probability of incorrectly accepting the hypothesis of statistical Control (ie a type II or β error) against the variable fraction non-confirming. The OC Curve provides a measure of the sensitivity of the Control Chart _ that is, its ability to detect a shift in the process fraction non confirming from the nominal value þ(t) to some other value þ(t). The probability of type II error may be computed from

$$\beta = P\{D < nUCL|p(t)\} - P\{D \leq nLCL|p(t)\}$$

Where D is a binomial variable and β – error defined in the above equation can be obtained from the cumulative binomial distribution. When the LCL is negative, the second term of the above equation should be dropped.

Illustration

Generate the OC curve for a control chart for variable fraction non-confirming with parameters

n = 50, UCL = .4602 and LCL = .2678

β = P {D < 50 (.4602)/ þ(t) } - P { D < 50 (.2678) / þ(t)}

= P {D < 11 / þ(t) } - P { D < 6 / þ(t)}

Table 1: Values of n = 50, UCL = .4602, LCL = .2678, w (t)-t/2=0.05

þ	þ(t)	P { D ≤ 19 / þ(t)}	P { D ≤ 1 / þ(t)}	β = P { D ≤ 19/ þ(t)} - P { D ≤ 1 / þ(t)}
.01	.011	1.000	.9106	.0894
.03	.032	1.000	.5553	.4447
.05	.053	1.000	0.2794	.7206
.10	.105	1.000	.0338	0.9662
.15	.158	0.997	0.0029	0.9951
.20	.21	0.9961	.0002	0.9959
.25	.262	.9956	.000	.9956
.30	.315	.8243	.000	.8243
.35	.367	.5912	.000	.5912
.40	.42	.3125	.000	.3125
.45	.473	.1485	.000	.1085
.50	.53	.0227	.000	.0227
.55	.578	.0041	.000	.0041

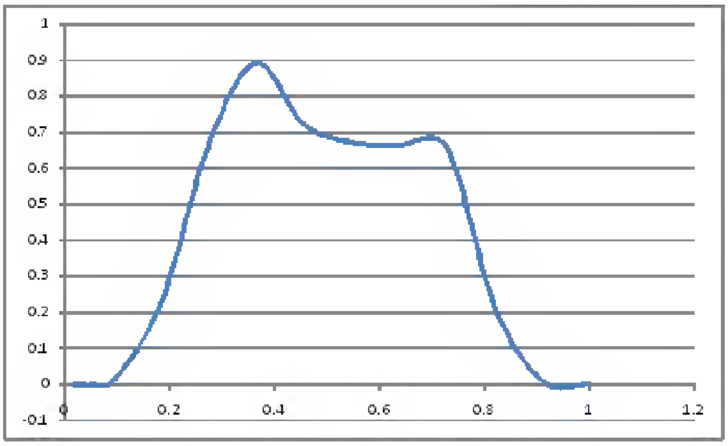

Figure 1: ARL Curve for VFD-P Chart

CONCLUSIONS

In this paper new control limits for p-chart are derived. ARL is determined and it is found that when the variation factor with respect to time is very small Shewart's p-chart and the proposed p-chart for variable fraction defective coincide.

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